Vortex Rollup Over and Behind Wings Related to Nonlinear Aerodynamic Characteristics

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Abstract

THE present paper describes the investigation of the rolling up of vortices over and behind wings of rectangular and delta planforms. The present method can also be used to evaluate the loading of these wings. The method is based on the representation of the vortex distribution over the planform by a vortex lattice system incorporating in this calculation the influence of the vortices shed from these cells into the flowfield. In this method, no a priori assumptions or restrictions are made concerning the shape of the vortex wake of the trajectories of the vortices. Therefore, this method can be applied to wing planforms of any aspect ratios. The evaluation of the diameter of the vortices far downstream of the wing, the spanwise position of the center of this vortex, and the appearance of the "secondary" vortices, which are some of the results obtained in this investigation, all are in reasonable agreement with available experimental results.

Contents

The interest in the evaluation of the characteristics of the flows involving vortices shed from lifting surfaces is due to two distinct features of these flows: 1) the effect due to the rolled up vortex sheet which results in the "tip" vortices, which are formed behind the lifting surfaces; 2) the "near field" effects due to the vortices that are shed from the planform edges and whose trajectories extend over the wing surfaces. The early investigations were concerned with vortices that are shed from high-aspect-ratio wings initially having elliptical distributions. Modification of these calculations to include the effects of the bound vortex and to include reasonable approximation to the chordwise and the spanwise lift distributions were also presented. The formulation of the Vortex Lattice Method, which represents each cell by a horseshoe vortex element, as presented in Refs. 1 and 2, enables the representation of vortex separation from all the edges or from any position on the surface by allowing the free vortices from each cell to separate locally into the freestream. The vortices that are separated are allowed to interact and to be set into the proper rolled up pattern. This flow pattern establishes a realistic vortex wake shape.

The numerical problems associated with the use of ideal line vortices lead us to look for a solution by the use of finite core vortices. Since the actual wake has thickness and is not an infinitesimally thin sheet, this is also a better representation of the real wake. We assume a core diameter determined by the condition of constant vorticity per unit cross-sectional area of the core. The density of the vorticity in the core can then be calculated, for instance, by using the method proposed by Spreiter and Sacks.[3] We impose the condition that, when any two vortex cores touch, the two vortices are replaced by a single one placed at the center of gravity of the pair. The new core diameter is determined by the condition of conservation of vorticity with the same vortex density. This procedure overcomes the difficulties due to the use of ideal vortices, i.e., "intersection" and "escape." This procedure also limits the number of vortices "automatically," thus helping to eliminate other irregularities in the computation. Furthermore, this method ensures that when all the vortices are amalgamated far downstream, we end up with a pair of vortices with the correct diameter and strength matching the expected vortex pair in the far field.

The calculation procedure follows these steps:

1) The wing planform is first divided into cells by the spanwise and chordwise divisions as in the standard vortex lattice method. One has to select a reasonable number of divisions. Each cell is then represented by a separate horseshoe vortex with the bound-vortex position at the cell's ¼-chord line and the trailing vortices are embedded in the wing planform and leave the wing at the trailing edge. The vortex strengths are calculated to be compatible with the boundary conditions applied at the control at the middle of the ¾-chord line of each cell. For rectangular wings the number of chordwise and spanwise divisions can be selected as required, whereas in the case of low-aspect-ratio delta wings, it was preferred to use an equal number of spanwise and chordwise divisions.

2) The induced velocities at each point of the wake vortex net are calculated using the Biot-Savart Law. The position of the downstream edge of each segment in the wake flowfield is calculated taking into account the induced velocities at the upstream edge. The effect of self-induced velocities on each vortex is accounted by using a concept of a cut-off radius.

3) The local position of the vortices can be evaluated from the calculated flow velocities:

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{v}{u+U}\qquad\frac{\mathrm{d}z}{\mathrm{d}x}=\frac{w}{u+U}$$

The Euler integration method is found to give reasonable results and no improvement was found by the use of the second-order Runge-Kutta technique.

4) Knowing the shape and position of the vortices the strengths of the vortices are then calculated by a solution of the matrix equation

$$\{\tan\alpha\}=|H|\{K\}$$

where $|H|$ is the influence matrix and $\{K\}$ is the matrix for the strength of the vortices while α is the local angle of attack. In the present Modified Vortex Lattice Method the natural interactions of every vortex segment are taken into account resulting in the evaluation of the actual trajectories. This calculation enables the evaluation of the rolled up vortex sheet characteristics behind the wing. The correct vortex wake shape is calculated by successive interactions using the new calculated vortex strength and the new shapes evaluated at each previous step.

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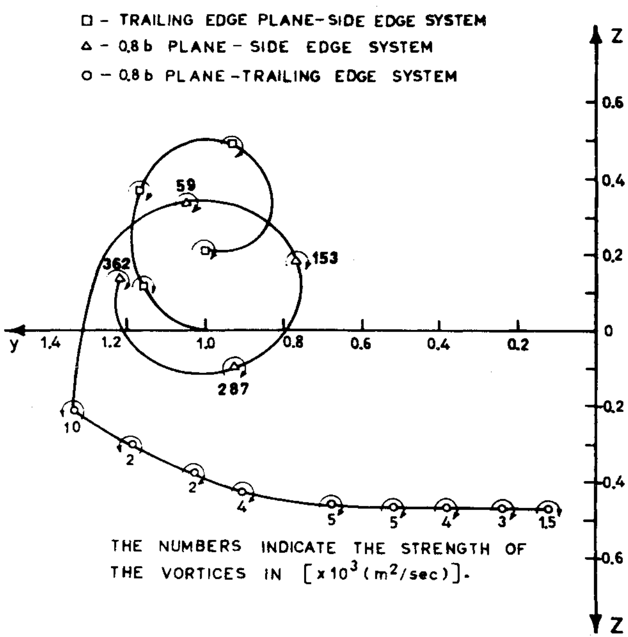

Fig. 1 The near-wake development behind a rectangular wing of Æ = 1 at $\alpha = 20$ deg.

Following the assumption that vortex separation may occur anywhere over the planform and in particular from the leading edges (or side edges in the case of a rectangular wing) the free vortices shed from each cell on the wing planform are allowed to follow the local flow direction. The initial trajectories of these vortices are selected to facilitate the convergence procedure. It is found that it is advantageous to select an initial condition such that the free vortices will leave the wing planform at an angle equal to half the local angle of attack.

Results of the Calculation of the Vortex Wake

Rectangular Wings

The spanwise position of the vortex core is calculated by the present method and the result is in good agreement with the experimental data presented in Ref. 4. In the case where the vortices are allowed to leave the wing planform at each cell, the calculation predicts the appearance of a secondary vortex system much weaker and of opposite sign to the main lift vortices. This result is a unique feature of the present method. The diameter of the vortex core far downstream can be evaluated from the calculated wake shape and can be compared with the experimental data of Ref. 4. The present calculation indicates a core size of $R_0/b = 2.7\%$ independent of angle of attack.

The developing shape of the trailing vortex wake behind a rectangular wing of $R = 1$, at $\alpha = 20$ deg is shown in Fig. 1. It can be seen that the vortex trajectories are such that for this slender rectangular wing there is a large vortex on the wing situated near each of the planform's side edges. The interaction of the rolled up side vortices with the vortices shed from the trailing edge is also shown in this figure. The rolled up vortex at the trailing edge already contains 42% of the total wing vortex strength. At the distance of $0.8b$ behind the wing this vortex is much larger in size and in strength. The vortices shed from the tailing edge are much weaker and those shed near the tips are of opposite sign to the vortices in the rolled up side vortex. Therefore, a cusp is obtained at their intersection.

Delta Wings

In the present calculation method, the triangular planform is divided into an equal number of chordwise and spanwise divisions, $N_c = N_s$. The triangular cells have their bound vortex on a line connecting the $\frac{1}{4}c$ point to the tip of the triangle. The control point is located on the median line at the $\frac{3}{4}c$ distance from the apex. Although the results are encouraging, there are more difficulties in the calculations of delta planforms than those for rectangular ones, particularly for slender delta wings with aspect ratios less than 1. These difficulties arise because of the very strong induced velocities due to the small distances between the discrete vortices on the slender delta wing using comparatively large number of cells. The present calculations indicate that good results can be expected using the present method for delta wings of aspect ratios larger than 1.[5,6]

The computations are performed on the IBM 370/168 Computer. The calculations of the flow over the rectangular wings require approximately 150-200 sec, whereas those for the delta wings require more iterations so that the computing time is extended to about 300 sec. The calculation of the full vortex wake trajectories downstream to 10 or more half-spans require computer time of up to 600-800 sec.

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